LECTURE SERIES–01

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B.Sc(H)–II

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PAPER–III PHYSICS (EMW)

Maxwell's electromagnetic field equation:

There are four fundamental equation of electromagnetism known as Maxwell's equations which may be written in differential form as

$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{D} &= \rho \quad \text{—— (a)} \\ \vec{\nabla}\cdot\vec{B} &= 0 \quad \text{—— (b)} \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \quad \text{—— (c)} \end{aligned}\right\} \text{—— (1)}$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \quad \text{—— (d)}$$

$\vec{\nabla}\cdot\vec{E} = 0$

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

and $\vec{\nabla}\times\vec{B} = \mu(\vec{J} + \epsilon\frac{\partial\vec{E}}{\partial t})$

Here $\vec{D}$ = electric displacement vecter in coul/m²

ρ = charge density in coul/m³

$\vec{B}$ = magnetic induction in wb/m²

$\vec{E}$ = electric field intensity in volt/m

and $\vec{H}$ = magnetic field intensity in A/m turn.

Each of maxwell's equation represents a generalisation of certain experimental observation Equⁿ (1a) represents the differential form of Gauss's Law in electrostatic which in turn derive from Coulomb's Law, Equⁿ (1b) represents Gauss's Law in magnetostatics which is usually said to represents the fact that isolated magnetic poles do not exist in our physical world. equⁿ (1c) represents differential form of Faraday's Laws of electromagnetic induction and finally equⁿ (1d) represents modification of Ampere's Law to include time varying field.

Thus it is clear that Maxwell's equation represent mathematical expression of certain experimental results.

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Derivation of Maxwell's equation:

(1) $div\,\vec{D} = \vec{\nabla}\cdot\vec{D} = \rho$

Proof: Let us consider a surface S bounding a volume V in a dielectric medium. In dielectric medium total charge consists of free charge plus polarisation charge. If ρ and ρ_0 are the charge densities of free charge and polarisation charge at a point in small volume element dV, then Gauss's Law can be expressed as

$$\int_S \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0}\int_V (\rho+\rho_0)\,dV$$

But polarisation charge density $\rho_p = -div\,\vec{P}$

$$\therefore \int_S \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0}\int_V (\rho - div\,\vec{P})\,dV$$

or $$\int_S \epsilon_0\vec{E}\cdot\vec{ds} = \int_V \rho\,dV - \int_V div\,\vec{P}\,dV$$

From Gauss's divergence theorem

$$\int_S \epsilon_0\vec{E}\cdot\vec{ds} = \int_V (\epsilon_0\vec{E})\,dV$$

$$\therefore \int_V (\epsilon_0\vec{E})\,dV = \int_V \rho\,dV - \int_V div\,\vec{P}\,dV$$

or $$\int_V div(\epsilon_0\vec{E}+\vec{P})\,dV = \int_V \rho\,dV \quad \text{— (2)}$$ where $\epsilon_0\vec{E}+\vec{P} = \rho$

But $\epsilon_0\vec{E}+\vec{P} = \vec{D}$ = electric displacement vecter

$\therefore$ equ^n (2) becomes

$$\int_V div(\vec{D}-\rho)\,dV = 0$$ Since this equation is true for all volumes, therefore the integral in this equ^n must vanish $\therefore div\,\vec{D} - \rho = 0$

or $$\boxed{\begin{aligned} div\,\vec{D} &= \rho \\ \text{or}\quad \vec{\nabla}\cdot\vec{D} &= \rho \end{aligned}}$$ — 1st equ^n — (3)

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(ii) $\vec{\nabla}\cdot\vec{B}=0$ or $div\,\vec{B}=0$

Proof: Since isolated magnetic poles and magnetic currents due to them have no physical significance, therefore magnetic lines of force in general are either closed curves or go off to infinity. Consequently the number of magnetic lines of force entering any arbitrary closed surface is exactly the same as leaving it. It means that the flux of magnetic induction $\vec{B}$ across any closed surface is always zero.

$\therefore \int_s \vec{B}\cdot\vec{ds}=0$ Using Gauss's divergence theorem to change surface integral into volume integral, we get $\int_v div\,\vec{B}\cdot dv=0$

As the surface bounding the volume is arbitrary, therefore this eq^n holds only if the integral vanishes

i.e $\therefore div\,\vec{B}=0$ or $\vec{\nabla}\cdot\vec{B}=0$ } 2nd eq^n

(iii) Curl $\vec{E}=-\frac{\partial\vec{B}}{\partial t}$ or $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

Proof: By Faraday's Law, we know that emf induced in closed loop is given by $e=-\frac{\partial\phi}{\partial t}=-\int\frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$ — (4)

Since the flux $\phi=\int_s\vec{B}\cdot\vec{ds}$, where S is any surface having the loop as boundary. The emf is the work done in carrying a unit charge completely arround the loop. Thus

$$e=\oint\vec{E}\cdot\vec{dl} \quad — (5)$$

Where $\vec{E}$ is the intensity of the field associated with induced emf $\therefore$ from eq^n (4) and (5), we get

$$\oint\vec{E}\cdot\vec{dl}=-\int_s\frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$$

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By Stoke's theorem line integral can be transformed into surface integral. $\therefore \int_s (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$

This integral is true for any surface whether small or large in the field

$$\therefore \boxed{\begin{array}{l} \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \end{array}} \quad \text{——— (6)} \rightarrow \text{3rd eq}^n$$

(iv) Curl $\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Proof: According to Ampere's circuital Law, the work done carrying a unit magnetic pole once round a closed arbitrary path linked with the current is expressed by

$\oint \vec{H}\cdot d\vec{l} = I \int \vec{J}\cdot d\vec{s}$ where integral on the right hand is taken over the surface through which the charge flow corresponding to the current I takes place.

Now changing the line integral into surface integral by Stoke's theorem $\int_s \text{curl } \vec{H}\cdot d\vec{s} = \int \vec{J}\cdot d\vec{s}$

$$\therefore \text{Curl } \vec{H} = \vec{J} \quad \text{— (7)}$$

But Maxwell's assumed that the definition of current density $\vec{J}$ is incomplete and hence he added the displacement current density $\vec{J}_d$ to find the total current $\vec{C}$

$$\therefore \vec{C} = \vec{J} + \vec{J}_d$$

$$\therefore \text{Curl } \vec{H} = \vec{C} = \vec{J} + \vec{J}_d \quad \text{— (8)}$$

or $\quad \text{div}\cdot\text{curl } \vec{H} = \text{div}(\vec{J} + \vec{J}_d) = 0$

$$\therefore -\text{div } \vec{J}_d = \text{div } \vec{J} = \text{div}\left(-\frac{\partial \rho}{\partial t}\right)$$

$$\therefore \text{div } \vec{J}_d = \frac{\partial \rho}{\partial t}$$

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Gauss's divergence theorem

$$\text{div}\,\vec{D} = \rho$$

$\therefore$ $\text{div}\,\vec{J}_d = \frac{\partial}{\partial t}(\text{div}\,\vec{D}) = \text{div}\frac{\partial \vec{D}}{\partial t}$ $\quad\therefore\quad \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$

$\therefore$ eqn (8) becomes

$$\boxed{\begin{aligned} \text{curl}\,\vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \\ \text{or}\quad \vec{\nabla}\times\vec{H} &= \vec{J} + \frac{\partial \vec{D}}{\partial t} \end{aligned}} \quad \text{—— (9)} \quad \rightarrow \text{4th eqn}$$

Physical Significance of Maxwell's equation:

(1) Maxwell's first equation is $\vec{\nabla}\cdot\vec{D} = \rho$

Integrating this over an arbitrary volume V, we get

$$\int_V (\vec{\nabla}\cdot\vec{D})\,dV = \int_V \rho\,dV$$

Changing volume integral of L.H.S in surface integral by Gauss's divergence theorem, we get

$$\int_S \vec{D}\cdot\vec{ds} = \int_V \rho\,dV \quad \text{—— (1)}$$

where S is the surface which bound the volume V. eqn represents Maxwell's 1st eqn $\vec{\nabla}\cdot\vec{D} = \rho$ in integral form.

Since $\int_V \rho\,dV = q$, the net charge contained in volume V.

The net outward flux of electric displacement vector through the surface enclosing volume is equal to the net charge contained within that volume.

(2) Maxwell's 2nd equation is $\vec{\nabla}\cdot\vec{B} = 0$

Integrating this eqn over an arbitrary volume, we get

$$\int_V \vec{\nabla}\cdot\vec{B} = 0$$

Using Gauss's divergence to change volume integral into surface integral, we get

$$\int_S \vec{B}\cdot\vec{ds} = 0 \quad \text{—— (2)}$$

where S is the surface which bounds the volume V. Eqn (2) represents

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Maxwell's 2nd equ^n in integral form

The net outward flux of magnetic induction $\vec{B}$ through any closed surface is zero.

(3.) Maxwell's 3rd equation is $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

Integrating this equ^n over a surface S bounded by a curve C, we get

$$\int_C (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Using Stoke's theorem in converting surface integral on L.H.S of this equ^n into line integral along the boundary of C, we get

$$\int_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{s} \quad \text{—— (3)}$$

This equ^n represents Maxwell's 3rd equ^n in integral form

The emf $\int_C \vec{E} \cdot d\vec{l}$ around a closed path is equal to negative rate of change of magnetic flux linked with the path.

(4.) Maxwell's 4th equation is $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$.

Taking surface integral over S bounded by curve C, we obtain

$$\int_S (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds$$

Using Stoke's theorem to convert surface integral on L.H.S of this equ^n into line integral, we get

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) ds \quad \text{—— (4)}$$

This equ^n represents Maxwell's 4th equ^n in integral form

The magnetomotive force ($mmf = \oint_C \vec{H} \cdot d\vec{l}$) around a closed path is equal to the conduction current plus displacment current through any surface bounded by the path.

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