## Magnitude of the bending Moment :

Let us consider a small part of the beam bent in the form of a circular arc, subtending an angle $d\theta$ at the centre of curvature O. Let $\rho$ be the radius of curvature of this part of the neutral axis NN' and

Let us consider a small filament a'b' at a distance z from the neutral filament ab. (fig.1)

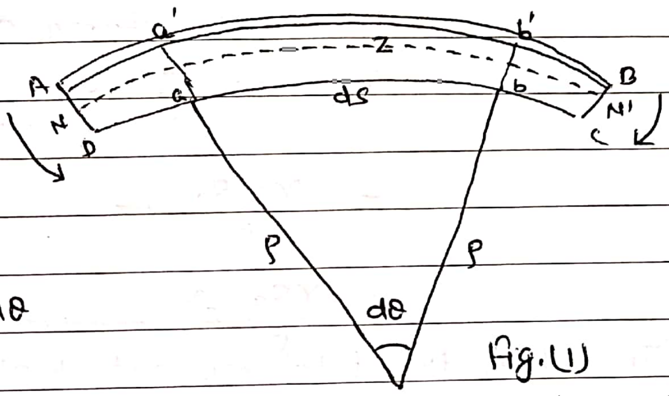


Fig.(1)

Then we have

$$a'b' = (\rho + z)\, d\theta$$

and $ab = \rho\, d\theta$

Now, before bending, the length of the filament a'b' was the same as that of ab (which remains unaltered on bending)

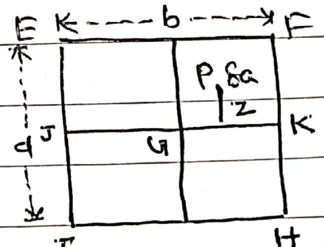
Hence, increase in length of the filament $a'b' = (\rho+z)d\theta - \rho d\theta$

$$= z\,d\theta$$

∴ The extensional or longitudinal strain

$$\frac{z\,d\theta}{\rho\, d\theta} = \frac{z}{\rho} \quad \text{——— (1)}$$

Now consider a section EFHI (fig.2) of the beam and suppose that the line JK lies on the neutral surface of the beam. The forces producing elongations act in the upper half EFKJ, and those producing contractions act in the lower half JKHI, in opposite directions perpendicular to the section EFHI. and hence constitute a couple.



We have first to find out the moment of this couple. The strain produced in a filament passing through a small area $\delta a$ around the point P distance z from

$$JK = \frac{z}{\rho}$$

Therefore, the tensile stress about P

$$= Y \times \frac{z}{\rho} \qquad \left[Y = \frac{\text{Stress}}{\text{Strain}}\right]$$

Where Y is young's modulus of the material of the beam.

Hence the force acting on the area

$$\delta a = Y \times \frac{z}{\rho} \times \delta a$$

The moment of this force about JK

$$= \frac{Y z \delta a}{\rho} \times z$$

$$= \frac{Y \delta a}{\rho} z^2 \qquad \text{——— (2)}$$

Therefore, the total moment of all such forces acting on the filaments of the whole section EFHI of the beam (bending moment about JK)

$$= \Sigma \frac{Y \cdot \delta a}{\rho} \cdot z^2 = \frac{Y}{\rho} \Sigma \delta a \cdot z^2$$

Summation being taken to cover all the filaments of the beam.

or Bending moment $= \frac{YI}{\rho}$.

Where $I = \Sigma(\delta a \cdot z^2)$ is a quantity analogous to moment of inertia and is called the geometrical moment of inertia of the cross section about the neutral axis, because here mass has been replaced by area.

Since at equilibrium, the restoring couple (i.e. bending moment) is equal to the moment of the external couple $\Gamma$ we have

$$\Gamma = \frac{YI}{\rho}$$

This is the required relation between the external couple $\Gamma$ acting on the section of the beam and the radius $\rho$ of the circular arc into which the beam is bent by the external couple.

From the geometry of the fig. It is evident that $\rho d\theta = ab = ds$ (say)

$$\frac{1}{\rho} = \frac{d\theta}{ds}$$

Where b is the breadth and d the thickness of the beam. ~~Hence equ~~$^n$

But $\theta = \tan\theta = \frac{dy}{dx}$ and as the curve is nearly flat at ab, $ds = dx$

$$\therefore \frac{1}{\rho} = \frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

$$\therefore \Gamma = \frac{YI}{\rho}$$

$$= YI\frac{d^2y}{dx^2} \quad \text{——— (3)}$$

(i) For a beam of rectangular cross-section, we have

$$I = bd^3/12$$

Where 'b' is the breadth and d the thickness of the beam

(ii) For a beam of circular cross section

$$I = \frac{\pi a^4}{4}$$

Where 'a' is the radius of the cylindrical beam

——— X ———

Teacher's Signature:________