DATE 17-04-20 | B.Sc(H)-II PAPER-III | PHYSICS OPTICS | By. Dr. M.K. THAKUR A.P.S.M. College Barauni

LECTURE SERIES-07

Fraunhofer's Diffraction at single slit

Let us suppose that a parallel beam of monochromatic light of wavelength λ is incident normally upon a narrow slit of width $AB = e$, as shown in figure 1. The slit is placed perpendicular to the plane of paper. The diffracted light is focussed by a convex lens L on a screen XY placed in the focal plane of the lens.

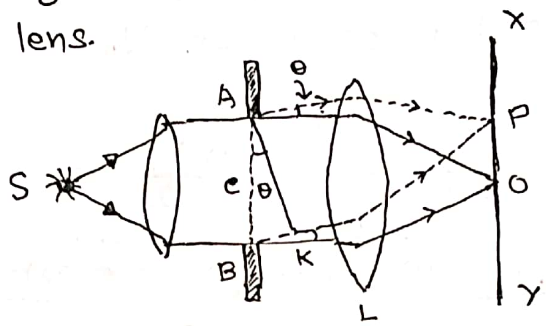

Fig.(1)

The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides.

According to Huygen's principle, plane wave front is normally incident on the slit AB, sends out secondary waveletes in all direction. The ray passes in same direction as the incident rays, are focused at O, the centre of screen. while those diffracted through an angle θ are focussed at P as shown in fig(1).

Intensity at P: Let AK be perpendicular to BK. As the optical paths from the plane AK to P are equal, the path difference between the wavelets from A to B in the direction θ is given by

$$BK = AB\sin\theta = e\sin\theta \quad [\because AB = e]$$

Hence the corresponding phase difference.

$$= \frac{2\pi}{\lambda} \text{ (path difference)}$$

$$= \frac{2\pi}{\lambda} (e \sin\theta)$$

Let us suppose that the width AB of slit is divided into n equal parts. Since the amplitude of vibration at P due to waves from each part will be the same. Let it be equal to a. Therefore the phase difference between the waves from any two consecutive parts is $\frac{1}{n}\left(\frac{2\pi}{\lambda} e \sin\theta\right) = d$ (say)

Hence the resultant amplitude at P is given by

$$R = \frac{a \sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)} = \frac{a \sin\left(\frac{\pi e \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin\theta}{n\lambda}\right)}$$

Let us put $\frac{\pi e \sin\theta}{\lambda} = \alpha$. Then

$$R = \frac{a \sin\alpha}{\sin \alpha/n} = \frac{a\sin\alpha}{\alpha/n} \quad \left[\because \frac{\alpha}{n} \text{ is very small}\right]$$

$$= na\left(\frac{\sin\alpha}{\alpha}\right)$$

As $n \to \infty$, $\alpha \to 0$, but the product na always remains infinite, Let $na = A$

$$\therefore R = A\left(\frac{\sin\alpha}{\alpha}\right)$$

Therefore the resultant intensity at P, being proportional to the square of the amplitude is

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{--- (1)}$$

The constant of proportionality is taken as unity for simplicity.

Maxima and Minima: From equ^n (1) that intensity is minimum (or zero) when $\frac{\sin\alpha}{\alpha} = 0$ or $\sin\alpha = 0$

But $\alpha \neq 0$, because when $\alpha = 0$, $\frac{\sin\alpha}{\alpha} = 1$.

$$\therefore \alpha = \pm m\pi \quad \text{——} \ (2)$$

Where m is an integer having values 1, 2, 3, ...

We have put earlier that $\alpha = \frac{\pi e \sin\theta}{\lambda}$. Therefore, equation (2) becomes

$$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

$$\text{or} \quad e\sin\theta = \pm m\lambda \quad \text{——} \ (3)$$

By putting m = 1, 2, 3, ---, we get the first, second, third, --- minima.

By differentiating equ^n (1) with respect to α and equating to zero, we can find the direction of maximum intensity. $\frac{dI}{d\alpha} = 0$

$$\text{or} \quad \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$\text{or} \quad A^2\left(\frac{2\sin\alpha}{\alpha}\right)\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

$$\text{or} \quad \alpha\cos\alpha - \sin\alpha = 0 \quad \therefore \alpha = \tan\alpha \quad \text{——} \ (4)$$

This equation is solved graphically, $y = \tan\alpha$

The first is a straight line passing through origin and making an angle 45°, while the second is discontinous curve having a number of branches as shown in fig (2). The point of intersection of the two gives the values of α satisfying equ^n ④ These values are approximately given out as

$$\alpha = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \text{------}$$

as more exactly as

$$\alpha = 0, 1.430\pi, 2.462\pi, 3.471\pi \text{---}$$

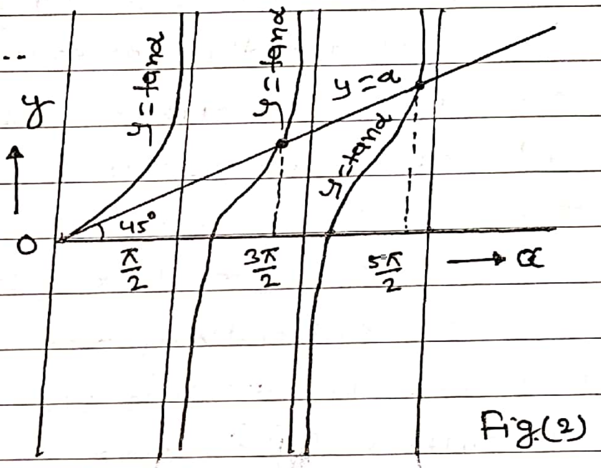

Fig.(2)

By Substituting the value of α in equ^n (1) we get the intensity of various maximum. Thus the intensity of the central maximum is

$$I_0 = A^2\left(\frac{\sin 0}{0}\right)^2 = A^2$$

Similarly, the intensity of first Subsidiary maximum is

$$I_1 = A^2\left\{\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right\}^2 = \frac{4}{9\pi^2}A^2 \approx \frac{A^2}{22}$$

that of the second Subsidiary maximum is

$$I_2 = A^2\left\{\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right\}^2 = \frac{4}{25\pi^2}A^2 \approx \frac{A^2}{61}.$$

Clearly, the intensity of successive maxima are the ratio

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2}$$

or $$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} \text{----}$$

Hence, most of the incident light is concentrated in the principal maximum. which occurs in the direction given by $\alpha = 0$ or $\frac{\pi e \sin\theta}{\lambda} = 0$ or $\theta = 0$

Which is in the direction of incident light.

Intensity curve:

Thus the diffraction pattern consists of a bright principal maximum in the direction of the incident light, having alternately minima and weak subsidiary maxima of rapidly decreasing intensity on the either side of it as shown in fig (3). The maxima lie at

$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi \pm \cdots$

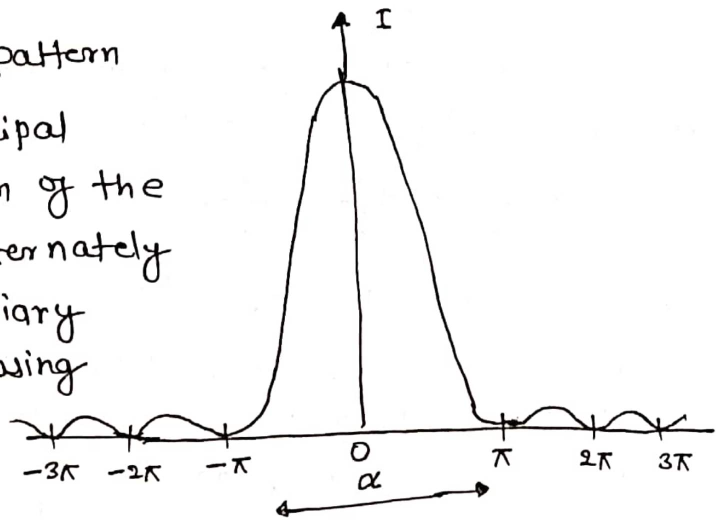

Fig-(3)

The subsidiary maxima do not fall exactly mid way between two minima but are displaced towards the centre of pattern by an amount which decreases with increasing order. If the collimating lens is very near the slit or the screen is far away from the lens and f = focal length of collimating lens, x = distance of the first minimum from O.

then
$$\sin\theta = \frac{\lambda}{e} = \frac{x}{f}$$

$$\therefore x = \frac{f\lambda}{e}$$

Hence, the width of the central maximum

$$= 2x = \frac{2f\lambda}{e}$$

—— x ——

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