B.Sc(H) PART-II * PHYSICS *

PAPER-III OPTICS

LECTURE SERIES-08

Date 18/04/2020

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Principle of Diffraction grating: A diffraction grating is a arrangement which is equivalent in action to a large number of parallel and equidistant slits of the same width. It is constructed by rulling equidistant parallel lines on a transparent material such as glass plate by means of a five diamond point worked with a ruling engine. This form a plane transmission grating. The number of lines on a plane transmission grating is of the order of 15000 per inch. If lines are ruled on a curved reflecting surface, it forms a concave reflection grating. The rulings scatter the light and are effectively opaque while the unruled parts transmit light and act as slits.

Theory: Let ABC--H be the section of a plane transmission grating supposed to be perpendicular to the plane of the paper as show in fig (1). Let the width of the transparent and opaque space or ruling be e and b respectively. The distance (e+b) is the grating element or grating constant. The point in two consecutive spaces, separated by e distance (e+b), are called corresponding points

Let us consider a parallel beam of light of wave length λ incident normally on the grating surface. Most of this light rays issuing from the spaces reach the point O on the screen XY in phase with each other, reinforce and produce a central maximum. A part of the light gets diffracted in various directions. Those diffracted at an angle θ with the initial direction reach P on passing through a convex lens L in different phases. As a result, dark and bright band on both sides of the central maximum are obtained.

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According to Hygen's principle as soon as the plane wave-front is incident on the slits, all points in each slit become the sources of secondary disturbance in all direction.

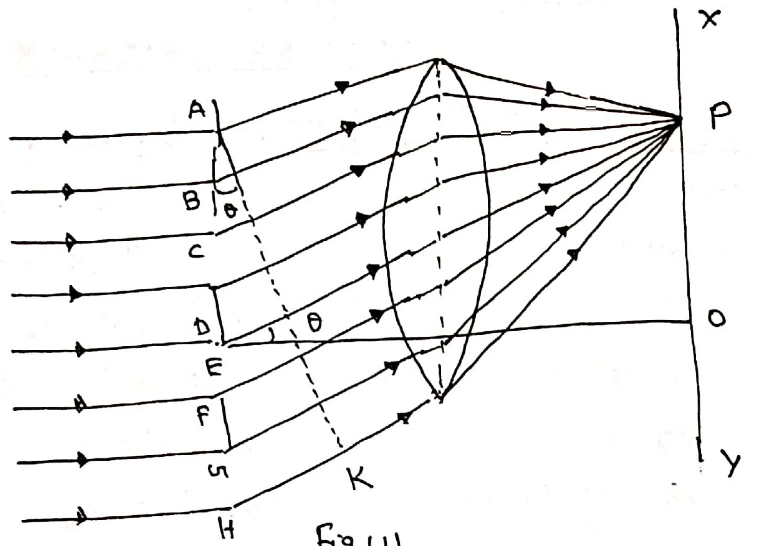

Fig.(1)

By the theory of Fraunhofer diffraction at a single slit, the secondary waves from all points in a slit diffraction in a direction θ are equivalent to a single wave of amplitude

$$R = A\left(\frac{\sin\alpha}{\alpha}\right)$$

where A is the amplitude due to a single slit in the normal direction α is the phase difference between the wavelets from the end and middle point of the slit i.e

$$\alpha = \frac{\pi}{\lambda}(e\sin\theta)$$

The resultant disturbance in each slit may be supposed to act at its middle point. Thus we have N diffracted parallel ray, one each from the middle points of the slits. Let us draw a perpendicular AN to CN. Then the path difference between the rays from the slits AB and CD is

$$CN = AC\sin\theta = (e+b)\sin\theta$$

Thus the path difference between the two rays from the corresponding points on two consecutive slits is $(e+b)\sin\theta$. Therefore the corresponding phase difference

$$= \frac{2\pi}{\lambda}(e+b)\sin\theta = 2\beta \text{ (say)}$$

Hence we may consider that there are N waves, each of amplitude R coming out from system of N slits and that there is a phase difference of 2β between each two successive waves. Therefore according to theorem of vector additions, we may write that the resultant amplitude to these waves will be given by

$$T = R \times \frac{\sin\frac{1}{2}(N.2\beta)}{\sin\frac{1}{2}(2\beta)} = A\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin N\beta}{\sin\beta}\right) \quad \text{—— (1)}$$

Hence the resultant intensity is given by

$$I = A^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{—— (2)}$$

This equation gives the distribution of intensity for finite number of beams with equal amplitudes and phases increasing in arithmatic progression. The facter $A^2 \sin^2\alpha/\alpha^2$ gives the intensity distribution in the diffraction pattern of single slit. The factor $\sin^2 N\beta/\sin^2\beta$ is supposed to represent the interference term and gives the distribution of intensity due to all the slits. Let Z denote this factor. The intensities of maxima and minima are obtained by equating its first differential co-efficient i-e $dZ/d\beta$ equal to zero. Now

$$Z = \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\therefore \frac{dZ}{d\beta} = \frac{\sin^2\beta \times 2N\sin N\beta\cos N\beta - 2\sin^2 N\beta\sin\beta\cos\beta}{\sin^4\beta}$$

$$= \frac{2N\sin N\beta\cos N\beta}{\sin^2\beta} - \frac{2\sin^2 N\beta}{\sin^2\beta}\frac{\cos\beta}{\sin\beta}$$

$$= \frac{2N\sin^2 N\beta\cos N\beta}{\sin N\beta\,\sin^2\beta} - \frac{2\sin^2 N\beta}{\sin^2\beta}\cdot\frac{\cos\beta}{\sin\beta}$$

$$= \frac{2\sin^2 N\beta}{\sin^2\beta}\left(N\cot N\beta - \cot\beta\right)$$

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For maxima or minima $\frac{dz}{d\beta} = 0$

$$\therefore \quad \frac{2\sin^2 N\beta}{\sin^2\beta}\left(N\cot N\beta - \cot\beta\right) = 0$$

Hence either $\frac{\sin N\beta}{\sin\beta} = 0$ ——— (3)

or $N\cot N\beta - \cot\beta = 0$ ——— (4)

(i) Principal maxima: Considering the equation $\frac{\sin N\beta}{\sin\beta} = 0$

If $\sin\beta = 0$, then $\beta = \pm n\pi$

where $n = 1, 2, 3, ---$.

Hence $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$, which is indeterminate. To find the value of this limit, the numerator and denominator are differentiated with respect to β

$$\therefore \quad \underset{\beta\to n\pi}{Lt} \frac{\sin N\beta}{\sin\beta} = \underset{\beta\to n\pi}{Lt} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$= \underset{\beta\to n\pi}{Lt} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

Thus the resultant amplitude in these direction is proportional to N, and hence the resultant intensity is proportional to N^2.

$$\therefore \quad I = T^2 = A^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)N^2; \quad I \propto N^2 \text{ ——— (5)}$$

These maxima are called principal maxima. The position of these maxima correspond to $\beta = \frac{\pi}{\lambda}(e+b)\sin\theta = \pm n\pi$ ——— (6)

or $(e+b)\sin\theta = \pm n\lambda$ where $n = 0, 1, 2, 3 - - - -$.

The $\pm$ sign indicates that there are two principal maxima of the same order lying on either side of the zero order maximum.

(ii) Secondary minima: When $\sin N\beta = 0$ but $\sin\beta \neq 0$, the factor $\sin N\beta / \sin\beta$ becomes zero and hence intensity is minimum.

Thus for minima, we have

$$\sin N\beta = 0 \quad \text{or} \quad N\beta = \pm m\pi$$

$$\text{or} \quad N\frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi \quad \text{——} \quad (7) \quad (\text{From equ}^n\ (6))$$

where m has all the integral values except $0, N, 2N, \ldots nN$ as for these values of m, $\sin\beta = 0$ and we obtain principal maxima. Thus it is evident that there are $(N-1)$ minima between two consecutive principal maxima.

(iii) Secondary maxima: As there are $(N-1)$ minima between two adjacent principal maxima. there must be $(N-2)$ other maxima known as secondary maxima, between two principal maxima. To find the position of secondary maxima, equ^n (4) becomes.

$$N\cot N\beta = \cot\beta$$

$$\text{or} \quad \tan N\beta = N\tan\beta \quad \text{——} \quad (8)$$

The roots of this equation other than those for which $\beta = \pm m\pi$ gives the position of secondary maxima. Its intensity is much less than the intensity of the principal maxima.

$\therefore$ From equ^n (8), we have

$$1 + \tan^2 N\beta = 1 + N^2\tan^2\beta$$

on solving, this gives rise to $\sin^2 N\beta = \dfrac{N^2\tan^2\beta}{1+N^2\tan^2\beta}$

$$\text{or} \quad \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta}{\sin^2\beta(1+N^2\tan^2\beta)}$$

$$\text{or} \quad \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\left(\frac{\sin^2\beta}{\cos^2\beta}\right)}{(1+N^2\tan^2\beta)\sin^2\beta}$$

$$= \frac{N^2}{\cos^2\beta + N^2\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta} \quad \text{——} \quad (9)$$

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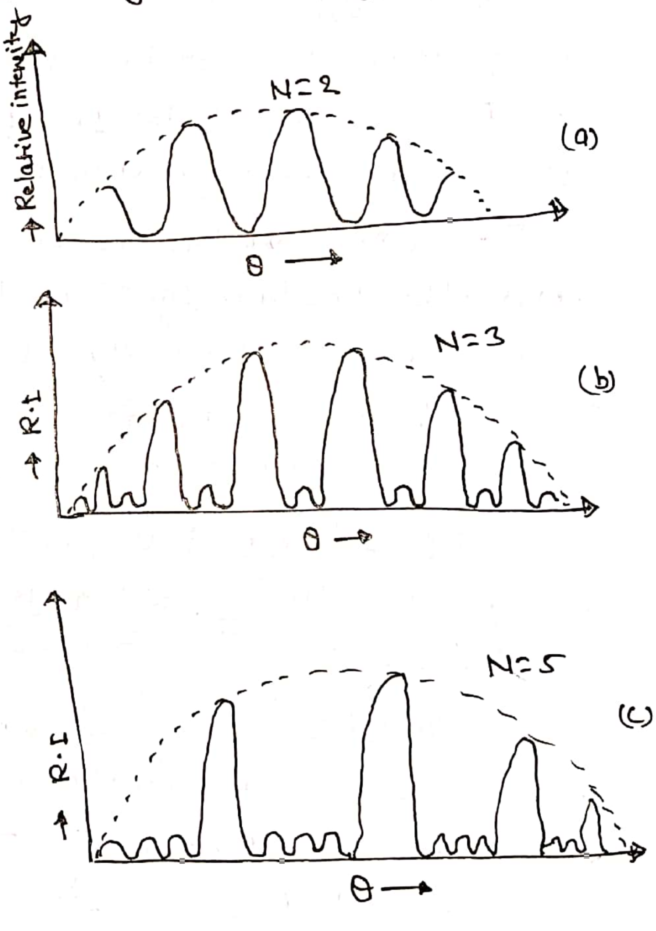
Which shows that the intensity of secondary maxima varies as $N^2/1+(N^2-1)\sin^2\beta$ while that of principal maxima varies as N^2. Hence the ratio of the intensity of these secondary maxima to the intensity of principal maxima is

$$\frac{1}{1+(N^2-1)\sin^2\beta}$$

So that larger the values of N, weaker are the secondary maxima. When N is large, as in usual practice, they have very week intensity and become invisible. When N is small they may be observed. As the number of slits increases, the number of secondary maxima also increases as shown in fig(2)a. Thus with two slits (N=2) we do not get any secondary maxima (fig 2a). With three slits (N=3), we get only one secondary maxima (fig 2b) and with five slits (N=5), we get three secondary maxima between two principal maxima as shown in fig(2c) It is evident from the figure that principal maxima become narrower as N increases.



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Determination of wavelength of light by grating:- From grating equation $(e+b)\sin\theta = n\lambda$. it is evident that if the grating element $(e+b)$ and the angle of diffraction θ for a particular order n be determind, the wavelength λ can be determined.

Determination of $(e+b)$: The grating element $(e+b)$ is determined from the number of rulings per inch on the grating. If this number is N, then

$$N(e+b) = 1 \text{ inch} = 2.54 \text{ cm}$$

$$\therefore (e+b) = 2.54/N \text{ cm}$$

Determination of θ: This is done with the help of a spectrometer. The slit of the spectrometer is illuminated by the given light, and the following adjustments are made.

(i) The eyepiece of the telescope is focussed on the cross-wires. (ii) The collimator and the telescope are adjusted for parallel rays by Schuster's method using a prism. (iii) The grating is adjusted on the prism table such that light from the collimator falls normally on it. To achieve this, the telescope is adjusted in position T_1, as shown in fig (3) such that the direct image of slit is seen at cross-wires of telescope. This makes the axis of the collimator. and the telescope in the same line. (iv) Now the telescope is turned through 90° in the position T_2. In this case the axis of the telescope is perpendicular to the axis of the collimator as shown in the figure. (v) Then the grating is mounted on turn table in position U_2 and the table is rotated till the image of the slit made by reflection of the light by the grating

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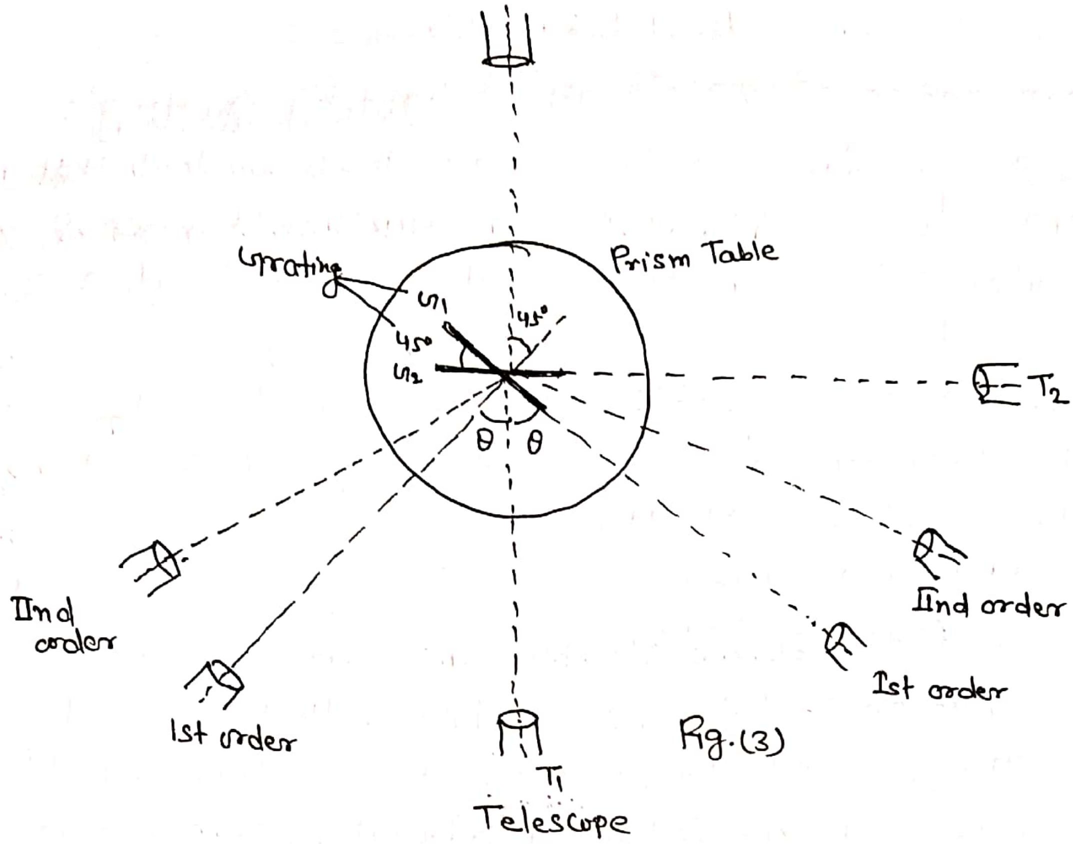

Fig. (3)

Surface, concides with cross wire of telescope. It is clear that in this position. the grating makes an angle of 45° with the axis of collimator or telescope. (VI) The table is now rotated through 45° so that the grating comes to position G_2 i.e it becomes normal to the incident light.

After this adjustment teleswpe is turned stil the first order spectrum is seen on either side of central position T_1. Then cross wire is set on different spectrum lines in turn and each time two varniers of spectrometer are read. The telescope is then turned to get the first order spectrum on other side and again the position of the spectral lines are read. It is evident from the figure that the difference between the two readings of any line on two sides give twice the angle of diffraction θ for that line in first order ($n=1$). Hence substituting $n=1$ and θ for different lines in grading eqn $(e+b)\sin\theta = n\lambda$, λ of different lines is calculated. How the same observation are repeated in 2nd order and λ can be-obtained

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