DATE 01-05-20 | B.Sc(H)-I | PAPER-I | * PHYSICS * | ELASTICITY | By. Dr. M.K. THAKUR | A.P.S.M. College, Barauni

**Elastic Limit:** Elastic limit is the maximum stress within which the body exhibits the property of elasticity. Below the elastic limit, the body regain its original position or shape or size when the deforming force is removed. Beyond the elastic limit, the body does not regain completely its original position or shape or size even though the external force is withdraw.

**Yield point:** When a wire is loaded beyond the elastic limit. Hooke's law is no longer obeyed and the extension produced in the wire is not proportional to the stress. Fig(1) Shows stress-strain curve for a wire. The extension is more than the corresponding increase in stress. At this stage, the particles of the material go further apart. Now, if the load is withdrawn, the particles do not regain their original positions. Point B in fig.(1) represent yieldpoint.

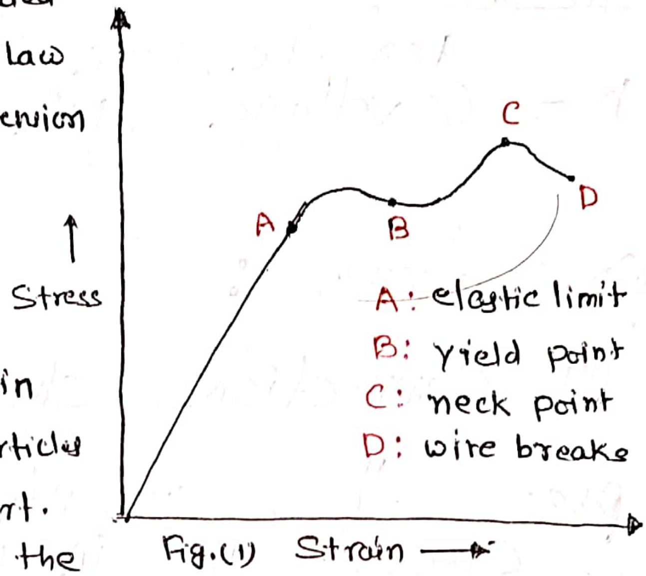



Fig.(1)

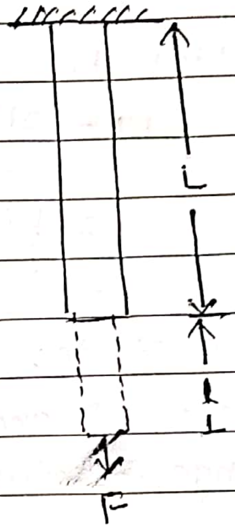
**Elastic Fatigue:** If a body continuously subjected to stress and strain after some period it gets fatigue. Consider two torsional pendulum A and B having similar wires. A is set into vibration. After A has come to rest, both A and B are set into vibration simultaneously. It is found that due to elastic fatigue, A comes to rest earlier than B.

**Work done in deforming a body:** When a body is deformed by the application of external forces, the body gets strained. The work done is stored in the body in form of energy and is called the potential energy.

We have seen that there are three types of strain (i) longitudinal (ii) shearing (iii) volume.

**(1) Longitudinal strain:** Let us consider a wire of length L, area of cross section A and young's modulus of elasticity Y. Let $l$ be the increase in length, when a streching force F is applied to the free end of the wire.

The work done for an increase dl is




$$dW = F\,dl$$

Hence, the work done

$$W = \int dW = \int_0^l F\,dl$$

If Y be the young's Modulus of the wire

$$Y = \frac{F/A}{l/L} \quad \text{or} \quad F = \frac{YA}{L} \cdot l$$

$$\therefore W = \int_0^l \frac{YA}{L} \cdot l \cdot dl = \frac{1}{2} \frac{YAl^2}{L}$$

$$= \frac{1}{2}\left(\frac{YAl}{L}\right) l = \frac{1}{2} F \cdot l$$

$\therefore$ Work done per unit volume

$$w = \frac{W}{V} = \frac{W}{AL} = \frac{1}{2}\frac{Fl}{AL}$$

$$w = \frac{1}{2}\frac{F}{A} \times \frac{l}{L} \quad \text{or} \quad \text{Work done} = \frac{1}{2}\,\text{Stress} \times \text{Strain}.$$

(ii) Shearing strain : Let us consider a cube of side of length L. When a tangential Force F is applied to the upper face of the cube keeping the lower face fixed, the cube is sheared through an angle $\theta$. If the tangential stress be T, we have

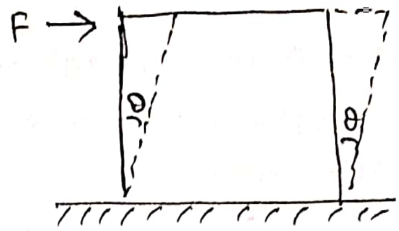
$T = F/A$ and modulus of rigidity.

$$\eta = \frac{T}{\theta} \quad \text{or} \quad T = \eta\theta$$

$\therefore$ Total tangential force $F = TA = A\eta\theta$

Now, the work done in shearing the cube through $d\theta$ by increasing the side dl is Fdl.

$\therefore$ Work done $= \int F dl$




But $d\theta = dl/L \quad \therefore dl = L \cdot d\theta$

$\therefore$ Total work done $= \int_0^\theta T (L d\theta)$

or $W = \int_0^\theta \eta A L \theta d\theta = \frac{1}{2} \eta A L \theta^2$

$\therefore$ Work done per unit volume,

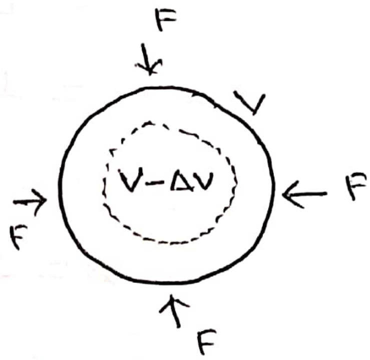
$$W = \frac{\eta A L \theta^2}{2AL} = \eta\theta^2/2 = (\eta\theta)\frac{\theta}{2}$$

$$= \frac{1}{2}(T)\theta$$

$= \frac{1}{2}$ shearing stress $\times$ shearing strain

(iii) Volume strain: Let the volume of a cube be $V$, area of cross-section $A$ and length $L$. When a normal stress $p$ is applied the change in volume is $v$

$\therefore$ Work done $= \int_0^v p dv$




If the bulk modulus for the material for the cube be $K$, we have,

$$K = \frac{pV}{v} \quad \therefore p = \frac{Kv}{V}$$

$\therefore$ Work done $W = \int_0^v \frac{Kv}{V} dv = \frac{1}{2}\frac{K}{V}v^2$

or $W = \frac{1}{2}\left(\frac{Kv}{V}\right)v = \frac{1}{2} p \times v$

Work done per unit volume

$$W = \frac{1}{2}\frac{p}{V} \cdot v = \frac{1}{2} p \cdot \frac{v}{V}$$

$= \frac{1}{2}$ volume stress $\times$ volume strain

## Effect of temperature and pressure on Elasticity:

(I) Effect of temperature: As a general rule modulus of elasticity decreases with temp. and for comparatively small ranges of temp. the relation is approximately linear. Lee and Share, and Andrews, worked at temp. to within 150°c of melting points of different materials and found that the relation between Y and T was of an exponential form $Y = Y_0 e^{-bT}$, where b has one value for temp. to about one-half the absolute temp. of the melting point, and another value for higher temp. For quartz Y has been found to change only slightly over the range 0°c to 800°c

Kohlrausch and Loomis conducted experiments for measuring the modulus of rigidity over a temp. range of 15°c to 100°c and expressed their results in the form $\eta = \eta_0(1 - \alpha T - bT^3)$ where $\eta$ is the modulus of rigidity at T°K and $\eta_0$ at absolute zero. Whereas a and b are constant for the materials concerned.

Bridgman found that the compressibility continuously decrease over the range from 0°c to 50°c. He expressed his results in the form $K = A + BT - CT^2$

(ii) Effect of Pressure: The bulk modulus K of water is a linear function of pressure at any given temp. Bridgman has found that At 0°c, $K = (2.02 + 0.000656\,P) \times 10^4$

At 50°C, $K = (2.36 + 0.000598\,P) \times 10^4$

The rigidity increased slightly under pressure, the ~~[illegible]~~ order of the change being about 2%, for an increase of pressure of 10,000 atmospheres. A decrease of rigidity with pressure has been found for glass, but both Young's modulus and Poission ratio increases about 3% for an increase in pressure of 10,000 atmosphere.

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Teacher's Signature :