Mathematics for Quantum Mechanism :-

(1) Function :-

a mathematical rule. It is simply denoted by

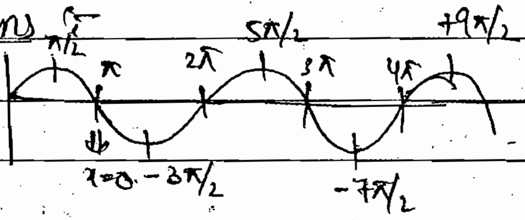
dependent variable $\Rightarrow y = f(x) \rightarrow$ variable independent

$\Downarrow$ depend on x.

Example $y = x$, $y = 3x$, $y = x^2$, $y = x^2 + 3$,

Some Important functions :-

① $y = \sin x$.



$\boxed{\sin n\pi = 0}$ $n = 0, 1, 2, 3.$

$y = \sin x$

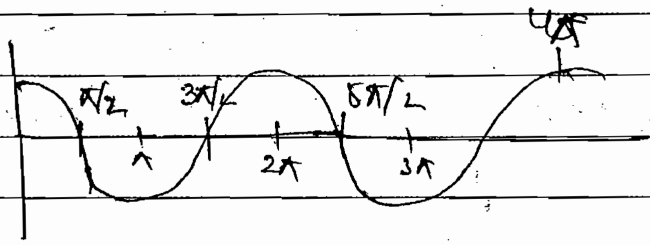
$\left.\begin{array}{l}\sin x = 0 \\ \sin x = +1 \\ \sin x = (-1)\end{array}\right]$ max, min

$\sin x = \sin(-x)$

$\sin x = -\sin x.$

$\boxed{y = f(x) = -f(x)}$

② $y = \cos x$.



$\left.\begin{array}{l}\cos 2\pi \\ \cos 4\pi \\ \cos 6\pi\end{array}\right] = +1$

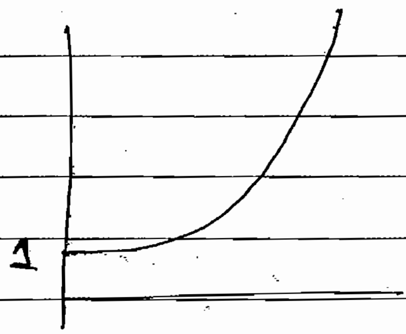
$\boxed{\cos 2n\pi = +1}$ $n = 0, 1, 2, 3 \ldots$

$\boxed{\cos \frac{n\pi}{2} = 0}$ $n = 0, 1, 2, 3 \ldots$

③ $y = e^x$ e = expon $\simeq 2.76$

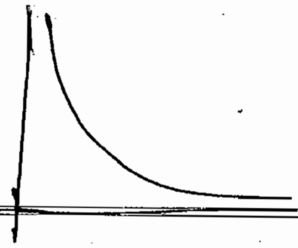
$y_{x=0} = e^0$

$\boxed{e^{\infty} = \infty}$ $\boxed{e^0 = 1}$ $\boxed{e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0}$



(4) $y = e^{-x}$

$$\boxed{e^{-0} = \frac{1}{0} = \infty}$$



(6) $y = mx$

(7) $y = mx^2 \Rightarrow$ parabola

(5) $y = x$ = straight line.

Algebric Relation :-

1) $(x+y)^2 = x^2+y^2+2xy$

(2) $(x-y)^2 = x^2+y^2-2xy$

3) $(x+y)(x-y) = x^2-y^2$

Trignometric Relations :-

$\Rightarrow$ particle in 1D & 2D box.

1) $\boxed{\sin^2 x + \cos^2 x = 1} \Rightarrow \sin^2 x = 1 - \cos^2 x$ or $\cos^2 x = 1 - \sin^2 x$

(2) $\boxed{\sin 2x = 2 \cdot \sin x \cdot \cos x}$

(3) $\boxed{\cos 2x = \cos^2 x - \sin^2 x}$

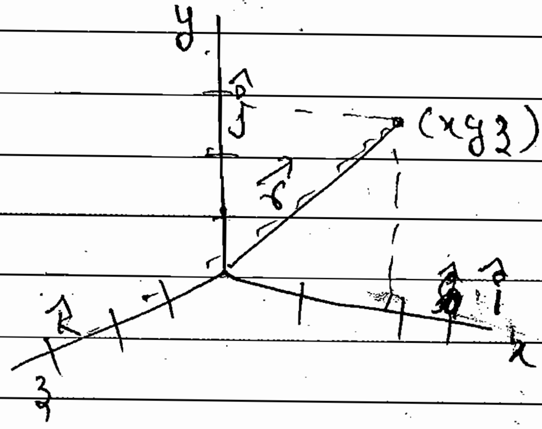
$\Rightarrow \cos^2 x - (1 - \cos^2 x)$

$\cos 2x = \cos^2 x - 1 + \cos^2 x \Rightarrow 2\cos^2 x - 1$

$$\boxed{\cos^2 x = \frac{1 + \cos 2x}{2}} \qquad \boxed{\sin^2 x = \frac{1 - \cos 2x}{2}}$$

Vector

$\Rightarrow$ angular momentum.



$\vec r = x\hat i + y\hat j + z\hat k$

$|\vec r| = \sqrt{x^2\hat i + y^2\hat j + z^2\hat k}$

$\vec p = p_x + p_y + p_z$

$\vec L = L_x + L_y + L_z$

$\hat i, \hat j, \hat k \Rightarrow$ unit vector along x, y, z.

$u = u_1 i + u_2 j + u_3 k$

$= b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$

cos $\boxed{\begin{array}{l} \hat{i} \times \hat{j} \\ = \cos 90° = 0 \end{array}}$ Multiplicat^n of vector

$\boxed{\hat{i} \times \hat{k} = \cos 90 = 0}$ ~~cos~~ dot (.) product | cross product

$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\Rightarrow$ (only $(\hat{i}\cdot\hat{i} + \hat{j}\cdot\hat{j} + \hat{k}\cdot\hat{k})$)

~~becos~~ ~~$a_1 \hat{i} + a_2 \hat{j}$~~

solution of determinant :-

$\Rightarrow$ order for multiplicat^n

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \Rightarrow a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$

$$\begin{vmatrix} 1 & 3 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{vmatrix} = 1(4-3) - 3(2-0) + 2(1-0)$$

$\Rightarrow 1 - 6 + 2$

$= -3$

$$\vec{a} \times \vec{b} \Rightarrow \begin{array}{ccc} + & - & + \end{array} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \Rightarrow \hat{i}(a_2b_3 - a_3b_2) - \hat{j}(a_1b_3 - a_3b_1) + \hat{k}(a_1b_2 - a_2b_1)$$

Q $\vec{a} = \hat{i} + \hat{j} + k$

$\vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$$\Rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{vmatrix}$$

$= \hat{i}(5-4) - \hat{j}(5-3) + k(4-3)$

$1\hat{i} - 2\hat{j} + k$

(1) $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} \Rightarrow x^2 - 1$

(2) $\begin{vmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{vmatrix} = x(x^2-1) - 1(x-0) + 0$

$x^3 - x - x = x^3 - 2x$

(3) $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = x(x^2-1) - 1(x-1) + 1(1-x)$

$= x^3 - x - x + 1 + 1 - x$

$= x^3 - 3x + 2$

Differentiatⁿ $\Rightarrow$ "change"

$y = f(x)$ representatⁿ $\Rightarrow \frac{dy}{dx}$ ← functⁿ, ← variable; $\frac{d}{dx}(f(x))'$

partial differentiation :- $\left(\frac{\partial}{\partial x}\right)$ $\Rightarrow$ In partial differentiatⁿ we only differentiate the functⁿ with respect to the only one variable the other parameters remain constant.

Integratⁿ is a reverse part of the differentiatⁿ.

Formulas :-

differentiatⁿ

1) $\frac{d}{dx}(x^n) = nx^{n-1}$

2) $\frac{d}{dx}\cos x = -\sin x$

3) $\frac{d}{dx}\sin x = \cos x$

4) $\frac{d}{dx}(e^x) = e^x$

5) $\frac{d}{dx}(\log x) = \frac{1}{x}$

6) $\frac{d}{dx}(a^x) = a^x \log_e a$

constant

$\boxed{\frac{d}{dx}[\text{cons}] = 0}$

(7) $\frac{d}{dx}\left(\frac{u}{v}\right) = \left(v\frac{du}{dx} - u\frac{dv}{dx}\right) \Big/ v^2$

Integratⁿ

(1) $\int f(x) \cdot dx$ ← variable

(2) $\int x^n = \frac{x^{n+1}}{(n+1)}$

(3) $\int \sin x \, dx = -\cos x$

(4) $\int \cos x \, dx = \sin x$

(5) $\int e^x \cdot dx = e^x$

(6) $\boxed{\int \frac{1}{x} \cdot dx = \log x.}$

use in thermodynamic.

(7)

Rules for differentiat.

1) $\frac{d}{dx}(c \cdot f(x)) \Rightarrow c\frac{d f(x)}{dx}$ ⇒ constant will be taken out of $\frac{d}{dx}$.

(2) $\frac{d}{dx}(u \pm v) \Rightarrow \frac{du}{dx} \pm \frac{dv}{dx}$ ⇒ + or − funct^n can be separated.

(3) $\boxed{\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}}$

Rules for Integrat^n

① & ② rule of differentiat^n is applicable in integrat^n also.

① $\int c f(x)\,dx = c\int f(x)\,dx$ ② $\int (u \pm v)dx = \int u\,dx \pm \int v\,dx$

③ $\boxed{\int \underset{\mathrm{I}\,\mathrm{II}}{uv}\,dx = \mathrm{I}\int \mathrm{II}\,dx - \int\left(\left(\frac{d\mathrm{I}}{dx}\right)\int \mathrm{II}\,dx\right)dx.}$

Q y find $\frac{dy}{dx}$ - ?

① $y = e^x + a^x + x^n$

$\frac{dy}{dx} = \frac{d}{dx}(e^x) + \frac{d}{dx}a^x + \frac{d}{dx}x^n$

$= e^x + a^x \log a + n x^{n-1}$

② $y = \sin x + \cos x$

$\frac{dy}{dx} = \frac{d}{dx}\sin x + \frac{d}{dx}\cos x$

$\cos x - \sin x.$

③ $y = x^2 \cdot e^x \cdot \sin x$

$\frac{dy}{dx} = e^x \sin x \frac{d}{dx}x^2 + x^2 \sin x \frac{d}{dx}e^x + x^2 e^x \frac{d}{dx}\sin x$

$= e^x \sin x(2x) + x^2 \sin x\, e^x + x^2 e^x \cos x$

$e^x x(2\sin x + x\sin x + x\cos$

④ $y = x + \frac{1}{x}$

$\frac{dy}{dx} = \frac{dx}{dx} + \frac{d}{dx}(1/x)$

$1 + (-1)x^{-1-1}$

$1 - x^{-2}$

⑤ $y = x^2 \cdot e^x$

$\frac{dy}{dx} = x^2\frac{d}{dx}e^x + e^x\frac{d}{dx}x^2$

$x^2 e^x + e^x(2x) \Rightarrow e^x x(x+2)$

⑥ $y = xe^x$

$\frac{dy}{dx} = x\frac{d}{dx}e^x + e^x\frac{d}{dx}x$

$xe^x + e^x$

$e^x(x+1)$

Q find $\frac{dy}{dx} = ?$

① $y = \sin(e^{x^2})$

$\frac{dy}{dx} = \cos(e^{x^2})(e^{x^2})(2x)$

$= 2xe^{x^2}\cos e^{x^2}$

② $y = \log(\sin x)$

$\frac{dy}{dx} = \frac{1}{\sin x} \times \cos x$

$\frac{\cos x}{\sin x} = \cot x.$

③ $y = \log(\sin x^2)$

$\frac{dy}{dx} = \frac{1}{\sin x^2} \cos x^2 \times (2x)$

$= 2x \frac{\cos x^2}{\sin x^2}$

$= 2x \cot x^2$

④ $y = \log(\log x^2)$

$\frac{dy}{dx} = \frac{1}{\log x^2} \times \frac{1}{x^{\cancel{2}}} \times 2\cancel{x} \Rightarrow \frac{2}{x \log x^2}$

⑤ $y = \sin ax$

$\frac{dy}{dx} = \cos ax \times (a)$

$= a\cos ax$

⑥ $y = e^{ax}$

$\frac{dy}{dx} = e^{ax} \times (a)$

$= a e^{ax}$

⑦ $y = \cos ax$

$\frac{dy}{dx} = -\sin(ax) \times a$

$= -a\sin ax$

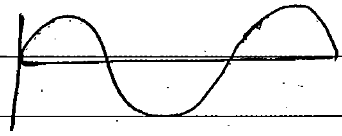
Condition of Maxima & Minima :-

~~process~~ :- steps :-

(1) find $y = f(x)$

② find $\frac{dy}{dx} = 0$ and solve for $x = a$ or b

③ find $\frac{d^2y}{dx}$



④ put $x = a$ or b in $\frac{d^2y}{dx}$

if $\left(\frac{d^2y}{dx^2}\right)_{x=a} = -ve \Rightarrow$ maxima.

$\left(\frac{d^2y}{dx^2}\right)_{x=a} = +ve \Rightarrow$ minima

Q find max & minima $y = x^3 - 3x^2 - 9$

$\frac{dy}{dx} = 3x^2 - 6x = 0 \quad = 0 \quad$ — ①

$x(3x - 6) = 0$

$\boxed{x = 2}$, $\boxed{x = 0}$

$\boxed{\frac{d^2y}{dx^2} = 6x - 6}$ — ②

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = 6\times 0 - 6 = (-6) = -ve = \text{funct}^n\ f(x) \text{ is } \underline{\text{maximum}}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=2} = 6\times 2 - 6 = 12-6 = +6 = (+ve) = \text{funct}^n \text{ is } \underline{\text{minimum}}$$

Q The angular momentum of a particle is defined as

$\vec{L} = \vec{r} \times \vec{p}$ $\quad \vec{r}$ = radius vector

find the L_x, L_y & L_z component of $\quad \vec{p}$ = linear momentum vector

the angular momentum.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

$$\begin{vmatrix} i & j & k \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix} = \vec{L} = \vec{r}\times\vec{p}$$

$$i(yp_z - zp_y) - j(xp_z - zp_x) + k(xp_y - yp_x)$$

$$iyp_z - zp_y i - xp_z j + zp_x j + \cancel{xp_z k} - \cancel{zp_x k}$$

$$i(yp_z - zp_y) + j(zp_x - xp_z) + k(xp_y - yp_x) \quad -(1)$$

$$\vec{L} = L_x i + L_y j + L_z \hat{k} \quad -(2)$$

comparing (1) & (2)

$$\boxed{L_x = yp_z - zp_y} \quad \boxed{L_y = zp_x - xp_z} \quad \boxed{L_z = xp_y - yp_x}$$

Q find the value of integrat^n.

(1) $\int \sin^2 x\, dx$ $= \cancel{\frac{1+\sin 2x}{2}}$ $\quad$ (2) $\int \cos^2 x\, dx$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\int\left(\frac{1-\cos 2x}{2}\right)dx = \int \frac{1}{2}dx - \frac{1}{2}\int \cos 2x$$

$$= \frac{1}{2}x - \frac{1}{2}\frac{\sin 2x}{2}$$

$$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right)$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \frac{1+\cos 2x}{2}dx$$

$$\int \frac{1}{2}dx + \int \frac{\cos 2x}{2}dx$$

$$\frac{1}{2}x + \frac{1}{2}\frac{\sin 2x}{2}$$

$$\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)$$

By part ... ... of integration :

I $\Rightarrow$ Inverse fx^n $\Rightarrow$ $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$

L $\Rightarrow$ logarithmic $funct^n$ $\Rightarrow$ $\log x$

A $\Rightarrow$ Algebric $funct^n$ $\Rightarrow$ x^2, x^3+1, x^2+2x etc.

T $\Rightarrow$ Trignometric $funct^n$ $\Rightarrow$ $\sin x$, $\cos x$, $\tan x$

E $\Rightarrow$ Exponential $funct^n$ $\Rightarrow$ e^x, $\cancel{x^2}\, e^{x^2}$ etc.

(3) $\int (ax^2+bx+c)\,dx = a\int x^2 dx + b\int x\,dx + c\int dx$

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx$$

$$\rightarrow x\left(\frac{ax^2}{3} + \frac{bx}{2}\right) + c)$$

(4) $\int \sin x \cdot \cos x \cdot dx \Rightarrow \cancel{\sin x \int \cos x\,dx + \cos x \int \sin x\,dx}$

$\cancel{\sin x\,(\sin x) + \cos x\,(-\cos x)}$

$\cancel{(\sin^2 x - \cos^2 x)}$

$\sin 2x = 2\sin x\cos x$

$\sin x\cos x = \frac{1}{2}\sin 2x$

$$\frac{1}{2}\int \sin 2x\,dx = \frac{-\cos 2x}{2\times 2} \times \cancel{\frac{2}{2}}$$

$$= \cancel{-\cos}\ \frac{-\cos 2x}{4}$$

(5) $\int \cancel{\sin n\pi}$

(6) $\int_0^a \sin^2\left(\frac{n\pi}{a}\right)x \cdot dx$

(b)

$\int_0^a \sin^2 bx \Rightarrow \left(\frac{1-\cos 2x}{2}\right) \Rightarrow \therefore \sin^2 bx = \frac{1-\cos 2bx}{2}$

$$\int_0^a \left(\frac{1-\cos 2bx}{2}\right) = \frac{1}{2}\int_0^a dx - \frac{1}{2}\int_0^a \cos 2bx\,dx$$

$$= \frac{1}{2}[x]_0^a - \frac{1}{2}\left(\frac{\sin 2bx}{2b}\right)_0^a$$

$\boxed{\therefore \int_0^a \sin^2 \frac{n\pi x}{a} = a/2}$

$$\frac{1}{2}(a-0) - \frac{1}{\cancel{2}4}\left(\sin 2\frac{n\pi}{a}x\right) \times \frac{a}{n\pi}\Bigg]_0^a$$

$$\frac{1}{2}(a-0) - \frac{a}{4n\pi}\left[\sin 2\frac{n\pi}{\cancel{a}}\times\cancel{a} - \sin 2\frac{n\pi 0}{a}\right]$$

$\boxed{\sin n\pi = 0}$ $\therefore \sin 2n\pi = 0.$

$$\frac{1}{2}a - \frac{a}{4n\pi}[\sin 2n\pi - \sin 0]$$

$$\frac{a}{2} - \frac{a}{4n\pi}[0-0] = \boxed{a/2}$$

9) $\int_0^a \sin\frac{n\pi x}{a} \cdot \cos\frac{n\pi x}{a}\, dx$.

$2\sin x\cos x = \sin 2x$

$\sin x\cos x = \frac{\sin 2x}{2}$.

$\frac{1}{2}\int_0^a \sin\frac{2n\pi x}{a}\, dx = \frac{1}{2}\left[\frac{-\cos\frac{2n\pi x}{a}}{2n\pi/a}\right]_0^a \Rightarrow \frac{-a}{4n\pi}\left[\cos\frac{2n\pi}{a} - \cos\right.$

$\boxed{\cos 2n\pi = +1}$

$\boxed{\cos 0 = +1}$

$= \frac{-a}{4n\pi}[\cos 2n\pi - \cos 0]$

$\frac{-a}{4n\pi}[1-1] \Rightarrow 0$. A

$$\boxed{\int_0^a \sin\frac{n\pi x}{a}\cos\frac{n\pi x}{a}\, dx = 0}$$

① $\int \underset{\text{I}}{x}\, \underset{\text{II}}{e^x} \Rightarrow x\int e^x dx - \int\left(\frac{d}{dx}x\int e^x dx\right)dx$

$= xe^x - \int 1 e^x dx$

$= xe^x - e^x \Rightarrow e^x(x-1)$ A

② $\int \underset{\text{I}}{x}\, \underset{\text{II}}{\sin x}\, dx$

$= x\int \sin x\, dx - \int\left(\frac{dx}{dx}\int \sin x\, dx\right)dx$

$= x(-\cos x) - \int 1(-\cos x)\, dx$.

$\Rightarrow -x\cos x + \sin x \Rightarrow \sin x - x\cos x$ A

③ $\int \underset{\text{I}}{x} \cdot \underset{\text{II}}{\sin^2 x}\, dx. \Rightarrow$