Charging of Condenser through Resistance (or CR-circuit)

Let us consider a circuit containing capacitor C, a resistor R, connected to source of emf through a key as shown in fig.(1). When the current is switched on by pressing Morse key the capacitor starts getting charged. Let q be the charge on the capacitor plates after a time t from start and let the charge grow at the rate $\frac{dq}{dt}$, then

potential difference across the capacitor plates $= \frac{q}{C}$

Potential difference across the resistor $= R \times i = R\frac{dq}{dt}$

If E is the applied emf, then applying Kirchhoff's 2nd law to the closed loop, we get

$$R\frac{dq}{dt} + \frac{q}{C} = E \qquad \text{--- (1)}$$

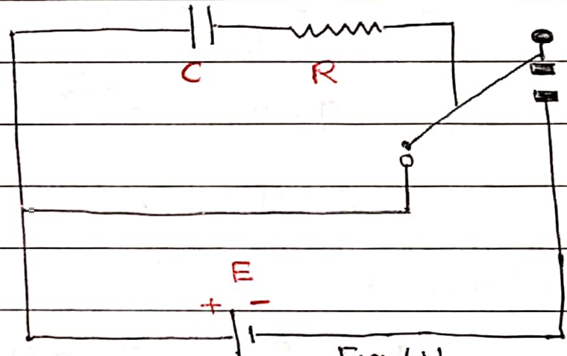

Fig.(1)

The capacitor plates continue to get the charge from source and when the potential difference across the capacitor plates is E, the emf, of the source the charge is maximum and constant $= q_0$

i.e $E = \frac{q_0}{C}$

From equ^n (1) becomes $R\frac{dq}{dt} + \frac{q}{C} = \frac{q_0}{C}$

or $q_0 - q = RC\frac{dq}{dt}$

or $\frac{dq}{q_0 - q} = \frac{1}{RC}dt$ Integrating the above equ^n, we get

$$\int \frac{dq}{q_0 - q} = \frac{1}{RC}\int dt$$

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$$-\log_e(q_0-q) = \frac{t}{RC} + A \quad \text{---(2)}$$

Where A is the constant of integration

When $t=0$, $q=0$

$$\therefore \; -\log_e q_0 = A$$

Substituting the value of A in eqn (2) we get

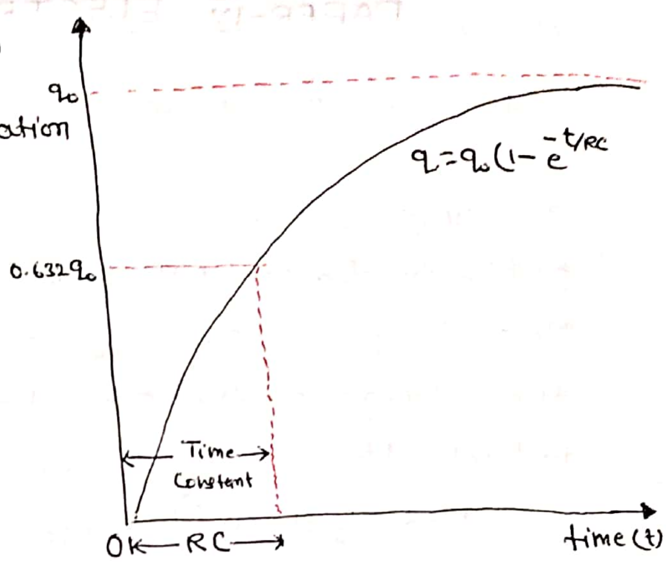
$$-\log_e(q_0-q) = \frac{t}{RC} - \log_e q_0$$

or $$\log_e\left(\frac{q_0-q}{q_0}\right) = -\frac{t}{RC}$$

or $$\frac{q_0-q}{q_0} = e^{\frac{t}{RC}}$$

or $$q_0-q = q_0^{-\frac{t}{RC}}$$

$$\therefore \; q = q_0(1-e^{-t/RC}) \quad \text{--- (3)}$$



Fig(2) Growth of charge in Condenser

Current

$$I = \frac{dq}{dt} = \frac{d}{dt}\left[q_0(1-e^{-t/RC})\right]$$

or $$I = \frac{q_0}{RC} e^{-t/RC}$$

The steady (maximum) current

$$I_0 = \frac{E}{R} = \frac{q_0}{RC}$$

$$\therefore \; I = I_0 e^{-t/RC} \quad \text{--- (4)}$$

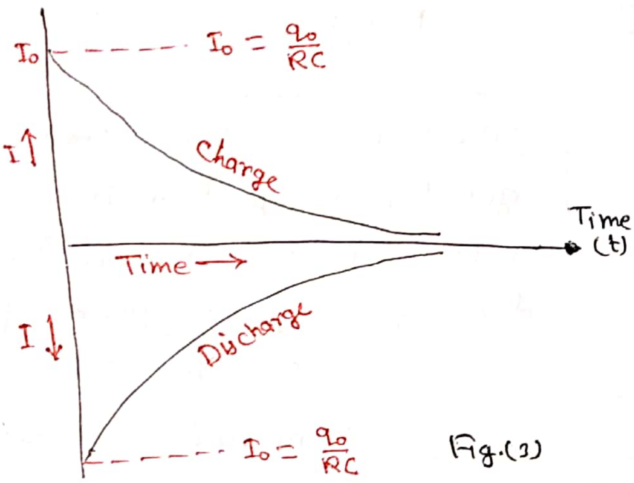


Fig.(3)

Variation of I during charging and discharging

The variation of charge with time at growth is shown in fig (2), Fig (3) shows the variation of current during charging and discharging of C through R.

Time constant. The time constant = RC (Capacitative time constant)

Taking $RC=t$, then $\frac{t}{RC} = 1$

$$q = q_0(1-e^{-1}) = q_0\left(1-\frac{1}{e}\right) = 0.6321 q_0$$

Hence, time constant RC is defined as the time taken by the capacitor to get charged to $(1-\frac{1}{e})$ or 0.6321 of the max^m value of charge.

Discharge of Capacitor through Resistance (R)

Let a capacitor C, having charge q_0 be connected to a resistance R. A current flows through the resistance. Let the charge left after a time t be q and let it decreases at the rate dq/dt.

then current after time t is $i = \frac{dq}{dt}$

$\therefore$ Fall of potential across the resistance $= Ri = R\frac{dq}{dt}$

and Fall of potential across the capacitor $= \frac{q}{C}$

Since there is no external source of emf, applying Kirchhoff's 2nd Law, we get $R\frac{dq}{dt} + \frac{q}{C} = 0$

or $\frac{dq}{dt} = -\frac{q}{RC}$ or $\frac{dq}{q} = -\frac{1}{RC}dt$

Integrating both side, we have

$$\log_e q = \frac{1}{RC}t + A \quad \text{—— (5)}$$

where A is a constant of integration.

When $t=0$, $q = q_0$ $\therefore A = \log_e q_0$

Substituting in eq^n (5) we get

$$\log_e q - \log_e q_0 = -\frac{1}{RC}t$$

or $\frac{q}{q_0} = e^{-\frac{t}{RC}}$ $\therefore q = q_0 e^{-\frac{t}{RC}}$ —— (6)

Time Constant. The time constant $= RC$

Taking $RC = t$, $\frac{t}{RC} = 1$

$$\therefore q = q_0 e^{-1} = \frac{q_0}{e} = 0.3679 q_0 \quad \text{—— (7)}$$

Teacher's Signature: ____________

Therefore, the time constant of a RC circuit may also be defined as the time during which the charge on the capacitors falls to $1/e$ or (0.3679) of its maximum value of charge.

The variation of charge during discharge of a capacitor with time is shown in fig(4)

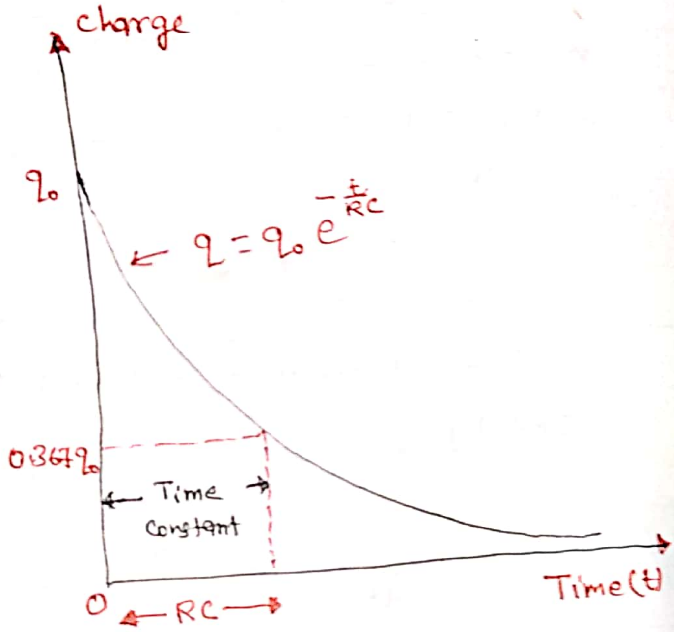
Current.

$$I = \frac{dq}{dt} = \frac{d}{dt}\left(q_0 e^{-t/RC}\right)$$

$$= \frac{-q_0}{RC} e^{-t/RC}$$

$$= -I_0 e^{-t/RC} \quad \text{——(8)}$$

where $I_0 = \frac{q_0}{RC}$ = the maximum current



Decay of charge of Condenser

At $t=0$, the current during charging is I_0 and during discharging is $-I_0$. The variation of I during charging and discharging of condenser is shown in fig (3)

The current I falls exponentially with time t

—— x ——